Fluid Flow and Heat Transfer in Micromachines

John Jones and Nilgoon Zarei, School of Engineering Science, Simon Fraser University

Summary ***The micromachined gyroscope invented by Dr Albert Leung is modelled using the COMSOL simulation package.***

Motivation

The `hot-air' gyroscope is a recent invention, and there are no established rules for optimizing its sensitivity and response time as a function of its design parameters. By creating and validating a numerical model of the gyroscope, we can readily explore a large design space and deduce rules for optimization.

Theory of Operation

The gyroscope is a cavity, approximately one cubic millimeter in volume, with silicon resistive heaters at both ends. By switching these heaters on and off alternately, we create an oscillating air flow across the cavity. If the gyroscope is rotating, Coriolis forces will divert this oscillating flow to one side, then the other. We place temperature sensors in the path of the diverted flow, measure the resulting temperature difference, and deduce the speed of rotation.

Governing Equations

We model the gas contained within the cavity as a system of five variables: temperature (T), pressure (p), density (ρ), and two components of velocity (u and v). One of the surprises in this work was that, although COMSOL supplies a wide range of pre-defined solution packages, none of these were suited to the problem. Instead, we created our own system of partial differential equations and used COMSOL's solvers to generate a solution.

Conservation of Energy.

$$\frac{c\rho}{k}\frac{dT}{dt} - \frac{d^2T}{dx^2} - \frac{d^2T}{dy^2} = -\frac{c\rho}{k}\left(u\frac{dT}{dx} + v\frac{dT}{dy}\right)$$

Conservation of Mass.

$$\frac{d\rho}{dt} + \frac{d\rho u}{dx} + \frac{d\rho v}{dy} = 0$$

Differential form of equation of state.

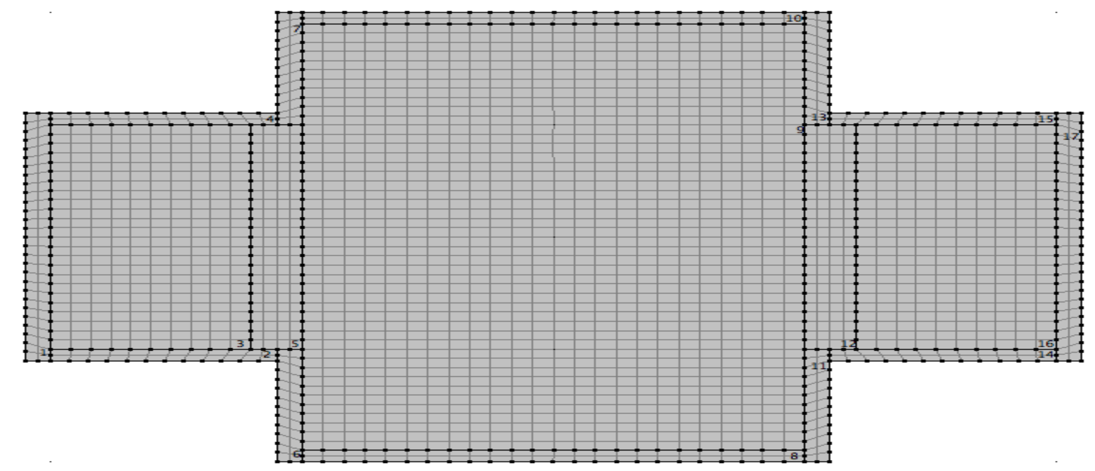
$$\frac{dp}{dt} - RT\frac{d\rho}{dt} - R\rho\frac{dT}{dt} = 0$$

Conservation of momentum, x-direction, Coriolis forcing term added.

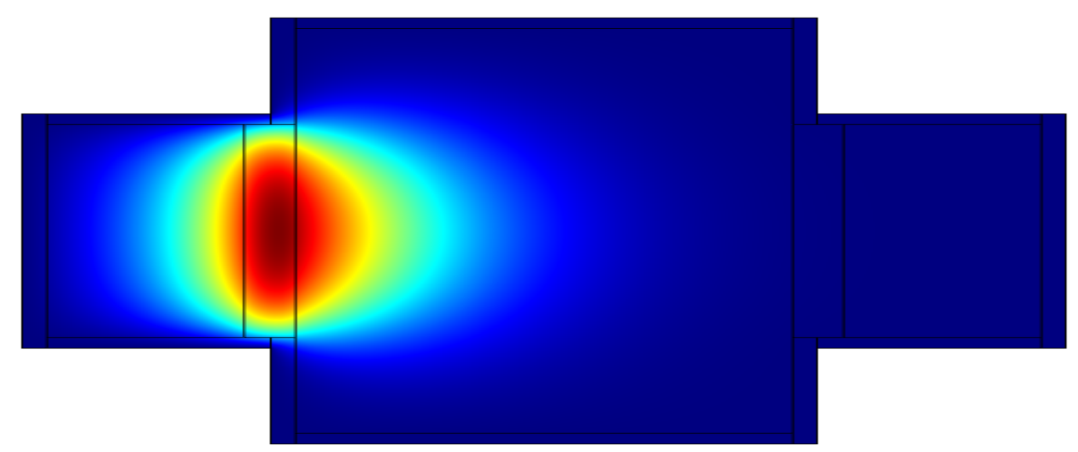
$$\rho\frac{du}{dt} + \frac{dp}{dx} - \mu\frac{d^2u}{dx^2} - \mu\frac{d^2u}{dy^2} = -2\rho v\omega$$

Conservation of momentum, y-direction, Coriolis forcing term added.

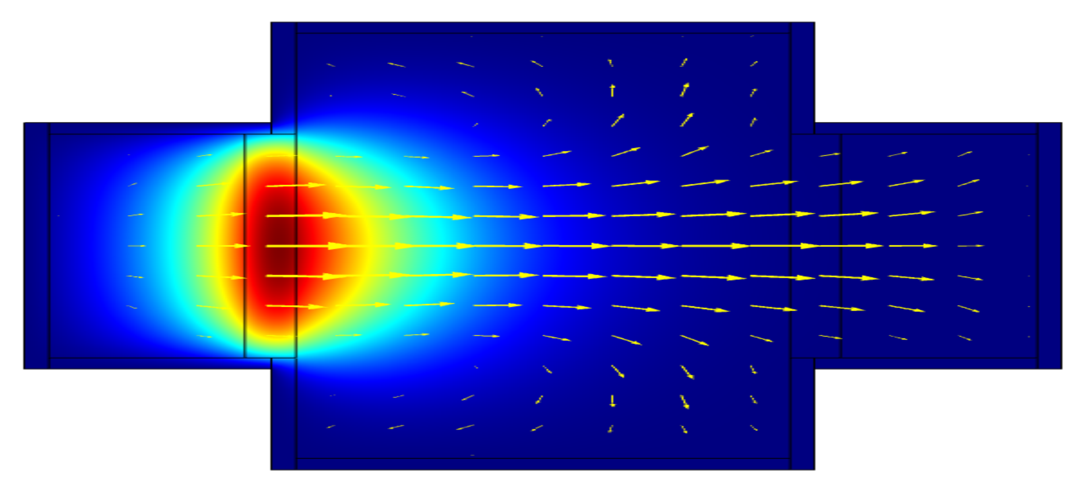
$$\rho\frac{dv}{dt} + \frac{dp}{dy} - \mu\frac{d^2v}{dx^2} - \mu\frac{d^2u}{dy^2} = -2\rho u\omega$$



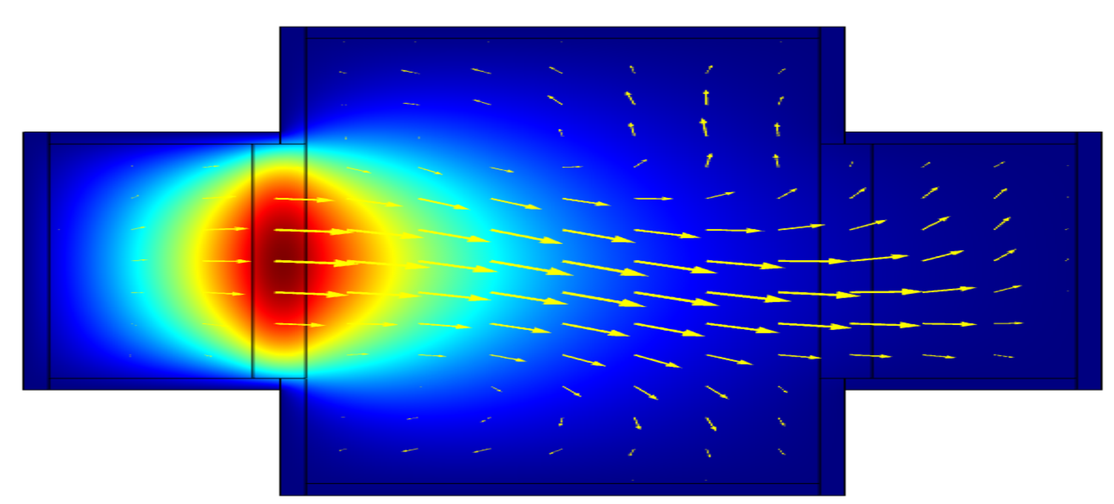
This is the mesh structure for the two-dimensional model used. Because it will be calculating a temperature *difference* of a few mK, it is essential that the mesh be symmetrical and the precision high. We refine the mesh until the calculated temperatures at symmetrically placed points agree to within a few μK.



As the heater switches on, the gas around it warms up.

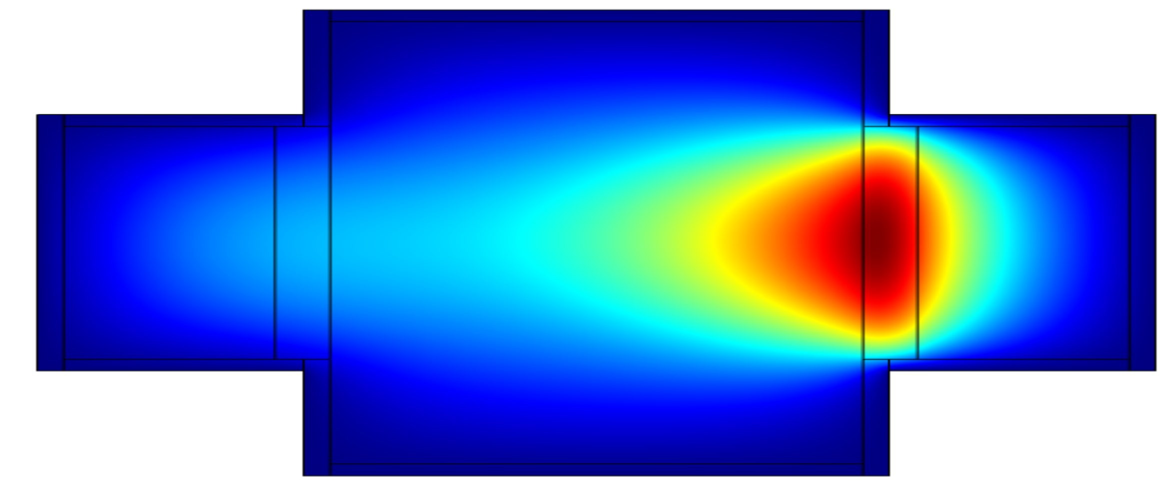


This creates a flow from west to east.

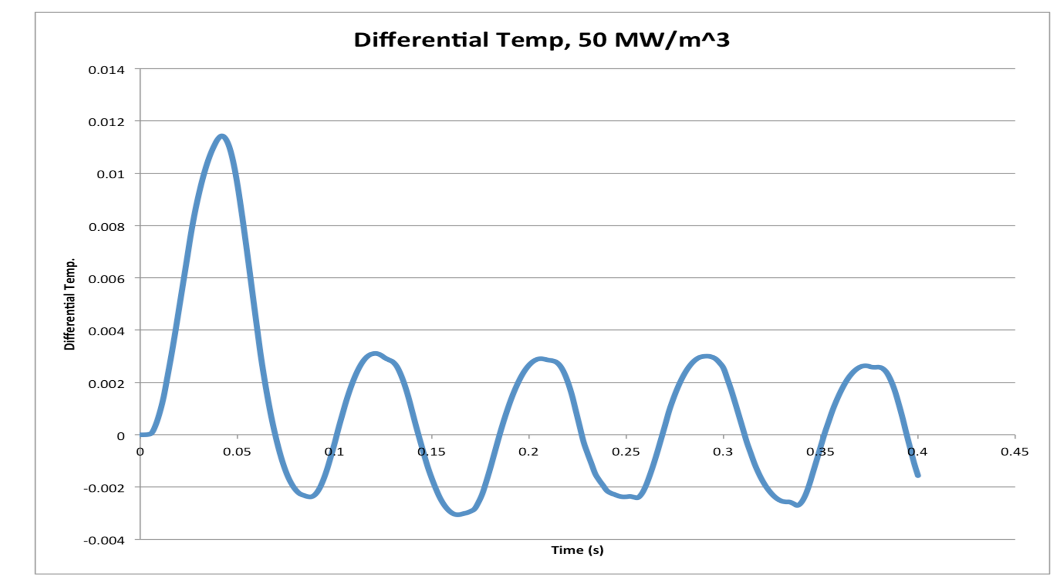


The Coriolis force diverts this flow to the south.

Then the other heater switches on



The cyclic temperature difference resulting from the diversion of the flow is too small to be seen from the colour plot, but we can record and graph it. The unusually high initial peak occurs as the gas first expands into a cold cavity.



Now we can use the model to compare a range of design options. In the figure below, we investigate the combined effects of changing the frequency at which we switch the heaters and the fraction of the cycle for which each heater is on. Although the model is too crude to match experiment exactly, it can predict general trends, as in the figure below.

At the next stage, we will create and tune a three-dimensional model.