Clearing House, Margin Requirements and Systemic Risk

Jorge Cruz Lopez[1], Jeffrey H. Harris[2], Christophe Pérignon[3].
[1]Simon Fraser University, Canada, [2]University of Delaware and Southern Methodist University, and [3]HEC Paris, France.

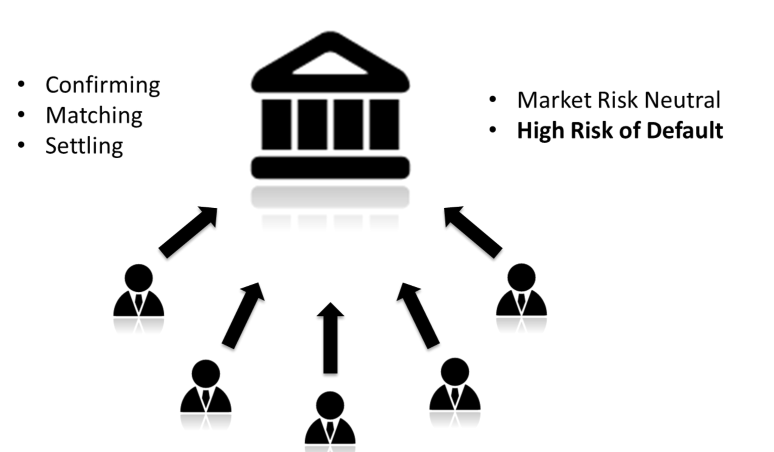
Objective

To propose a **more resilient margining system** for derivatives exchanges that accounts for ***joint financial distress.***

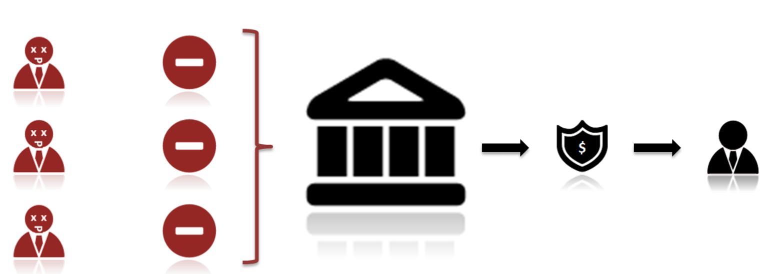
Abstract

Margins are the major safeguards against default risk on a derivatives exchange. When the clearing house sets margin requirements, it does so by only focusing on individual clearing member (CM) positions (e.g. the SPAN system). We depart from this traditional approach and present an alternative method that accounts for interdependencies among clearing members when setting margins. Our method generalizes the SPAN system by allowing individual margins to increase when clearing firms are more likely to be in financial distress simultaneously.

Derivatives Exchange



Systemic Risk



Methodology

- **Relative Variation Margin:** $R_{i,t} = V_{i,t} / B_{i,t-1}$
- **Financial Distress:** $B_{i,t-1} + V_{i,t} < 0 \rightarrow R_{i,t} < -1$
- **Tail Dependence:** Probability of two random variables having simultaneous extreme events in the same direction. We focus on the lower tail:

$$\tau^L_{i,j} = \lim_{\alpha \to 0} Pr\left[R_i \le F_i^{-1}(\alpha) | R_j \le F_j^{-1}(\alpha)\right]$$

- **Trading Revenue Dependence Modeling:** Estimate a *t-copula* using a two-stage semiparametric approach (Genest, Ghoudi, and Rivest 1995).

$$F(R_i, R_j) = C\left(F_i(R_i), F_j(R_j)\right)$$

$$T_{\rho,\upsilon}\left(F_i(R_i), F_j(R_j)\right) = t_{\rho,\upsilon}(R_i, R_j)$$

$$\tau_{i,j} = 2t_{\upsilon+1}\left(-\sqrt{\upsilon+1}\sqrt{(1-\rho)/(1+\rho)}\right)$$

  - **First Stage:** Estimate empirical marginal distributions.
  - **Second Stage**: Estimate, ρ and υ, through maximum likelihood.

Standard Collateral (B)

$$Pr\left[\tilde{V}^s_{i,t+1} \le -B_{i,t}\right] = q\ ;\ s = 1, \dots, S.$$

- CM i positions at time t: $w_{i,t} = \left[w_{i,1,t} \ \dots \ w_{i,D,t}\right]$.
- Consider S scenarios based on potential one-day ahead changes in the value (ΔX) and volatility ($\Delta\sigma_X$) of the underlying assets, as well as in the time to expiration of the derivatives products. For each of the S scenarios, we revaluate the portfolio (i.e. mark-to-model its positions) and compute the associated hypothetical P&L or variation margin on the portfolio: $\tilde{V}_{i,t+1} = \left[\tilde{V}^1_{i,t+1} \ \dots \tilde{V}^S_{i,t+1}\right]$.

Tail-Dependent Collateral (B*)

$$B^*_{i,t} = B_{i,t} \times e^{max\{\gamma(\tilde{\tau}_{i,t} - \underline{\tau});0\}}$$

- Consider the portfolios of derivatives contracts of two clearing firms at the end of a given day. For each clearing firm, we compute the variation margins generated by the S scenarios described in the previous section and calculate $B_{i,t}$ and $B_{j,t}$.
- The tail dependence between the clearing firms' simulated relative variation margins is given by:

$$\tilde{\tau}_{i,j,t} = \lim_{\alpha \to 0} Pr\left[\tilde{R}_{i,t+1} \le F^{-1}_{i,t+1}(\alpha) | \tilde{R}_{j,t+1} \le F^{-1}_{j,t+1}(\alpha)\right]$$

where $\tilde{R}_{i,t+1} = \tilde{V}_{i,t+1} / B_{i,t}$

- For each clearing firm we consider $\tilde{\tau}_{i,t} = max\left\{\tilde{\tau}_{i,j,t}\right\}^N_{j=1,j \ne i}$.
- γ is the tail-dependence aversion coefficient and $\underline{\tau}$ is a threshold tail dependence coefficient below which the collateral is not affected, i.e., $B^*_{i,t} = B_{i,t}$ if $\tilde{\tau}_{i,t} \le \underline{\tau}$.
- Thus, B* accounts for the **magnitude and dependence** structure across CMs' simulated losses.

Budget Neutral Collateral (B^0)

$$B^0_{i,t} = B_{i,t} + \frac{B^*_t - B_t}{n} \quad for\ \ i = 1, \dots, n$$

such that $\sum_{i=1}^n B^0_{i,t} = \sum_{i=1}^n B^*_{i,t}$

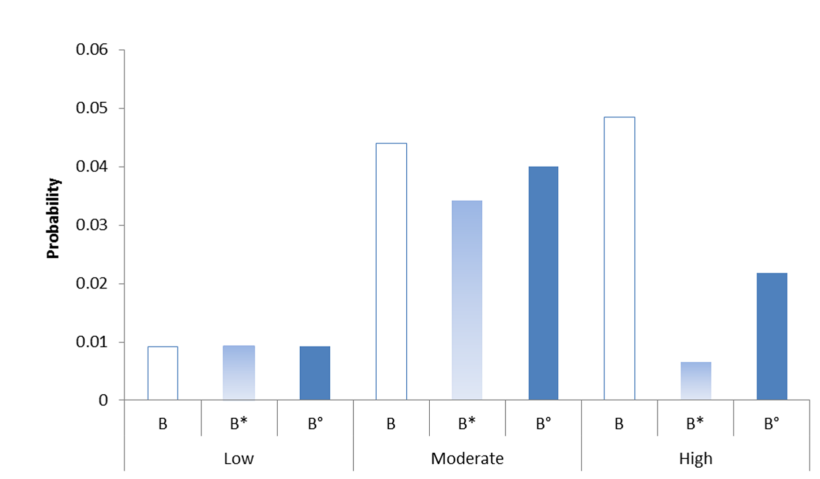
where $B_t = \sum_{i=1}^N B_{i,t}$ and $B^*_t = \sum_{i=1}^N B^*_{i,t}$

- Notice that B^0 provides a better benchmark against which to compare the tail-dependent margining system because it collects the same aggregate collateral.

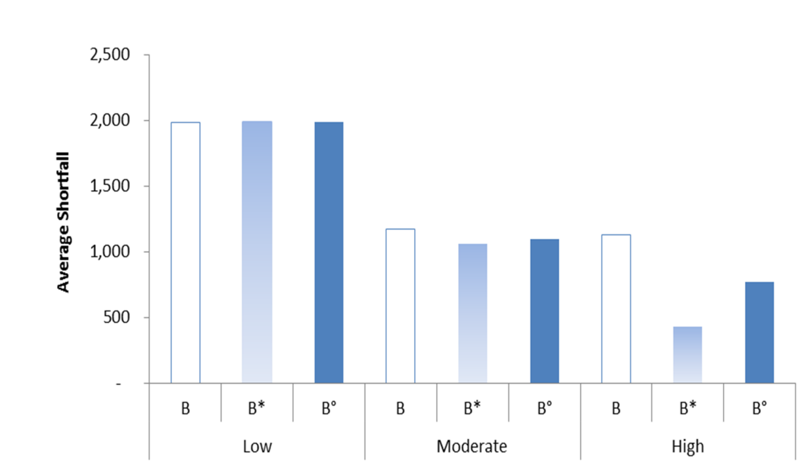
Controlled Experiment

	Low Tail Dependence				Moderate Tail Dependence				High Tail Dependence			
	1	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Panel A: Trading Positions												
$d = 1$	100	30	-50	-100	100	125	-50	-100	100	95	-50	-100
$d = 2$	100	-170	150	-100	100	75	150	-100	100	105	150	-100
Panel B: Tail Dependence Coefficients												
$\tilde{\tau}_{2,j}$	.000	.	.	.	.247	.	.	.	.908	.	.	.
$\tilde{\tau}_{3,j}$	.000	.000	.	.	.000	.000	.	.	.000	.000	.	.
$\tilde{\tau}_{4,j}$	.000	.000	.000	.	.000	.000	.000	.	.000	.000	.000	.
$\tilde{\tau}_i$	.000	.000	.000	.000	.247	.247	.000	.000	.908	.908	.000	.000
Panel C: Margins												
B_i	3,849	6,228	4,310	5,319	3,849	3,918	4,310	5,319	3,849	3,851	4,310	5,319
B^*_i	3,849	6,228	4,310	5,319	4,022	4,094	4,310	5,319	4,905	4,908	4,310	5,319
B^0_i	3,849	6,228	4,310	5,319	3,936	4,005	4,397	5,406	4,377	4,380	4,839	5,847
p_i	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
p^*_i	.050	.050	.050	.050	.041	.038	.050	.050	.007	.007	.050	.050
p^0_i	.050	.050	.050	.050	.045	.044	.046	.046	.022	.022	.026	.033

Probability of Joint Financial Distress



Shortfall Given Joint Financial Distress



Conclusion

The **tail-dependent margining system** is a new approach to compute margin requirements for a portfolio of derivatives securities that accounts not only for individual risk, but also for the **interdependence across CMs.**

Interdependence is measured through a simulation-based technique that accounts for tail dependence across CMs' potential trading losses. Thus, margin allocation is a function of the homogeneity of trading positions across CMs.

Our proposed system is superior to others because it provides a **better allocation of margin requirements** and it provides **better protection against joint negative outcomes.**